ANALYSIS OF A NONUNIFORM GAS FLOW PAST A SPHERE

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We present some results of analysis for the nonuniform flow of a gas past a sphere using the method of Dorodnitsyn-Belotserkovskii [1 to 3].

1. Let there be a nonuniform supersonic flow directed against a sphere (Fig.1); the flow is symmetric with respect to the x-axis and is given as

$$w_+ = f_1(\theta, r), \qquad \vartheta = f_2(\theta, r) \qquad (x = -r\cos\theta, \quad y = r\sin\theta) \tag{1.1}$$

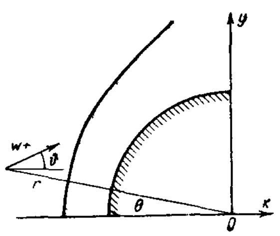

Fig. 1

Here w_+ is the magnitude of the velocity in the nonuniform flow, ϑ is the inclination of the velocity with respect to the axis of symmetry, f_1 and f_2 are continuous functions of the spherical coordinates θ and r. It is required to determine the shape and position of the shock wave and the mixed flow in the region of influence.

Let us refer the velocity w to the critical velocity a_*, the density ρ to the stagnation density of the unperturbed flow ρ_0, the pressure P to $\rho_0 a_*^2$, and the linear dimensions to the radius of the sphere. Then, using the Bernoulli integral and the stream functions ψ, the system of equations of gas dynamics in spherical coordinates may be written [2] in the following form:

$$\frac{\partial}{\partial r}[r^2(P + \rho u^2)\sin\theta] + \frac{\partial}{\partial\theta}[r(\rho uv)\sin\theta] = r(2P + \rho v^2)\sin\theta \tag{1.2}$$

$$\frac{\partial}{\partial r}(r^2\tau u\sin\theta) + \frac{\partial}{\partial\theta}(r\tau v\sin\theta) = 0, \qquad \frac{d\psi}{d\theta} = r\rho\left(v\frac{dr}{d\theta} - ru\right)\sin\theta, \qquad \varphi = \varphi(\psi)$$

$$P = \frac{\varkappa+1}{2\varkappa}\left(1 - \frac{\varkappa-1}{\varkappa+1}w^2\right)\rho, \quad \rho = \left(\frac{\varkappa+1}{2\varkappa}\right)^{1/(\varkappa-1)}\varphi^{1/(1-\varkappa)}\tau, \quad \tau = \left(1 - \frac{\varkappa-1}{\varkappa+1}w^2\right)^{1/(\varkappa-1)}$$

Here $\varphi = P/\rho^\varkappa$ is the entropy function, and u and v are the velocity components along r and θ. The unknown functions are u, v, φ, ψ. Let ε denote the distance from the sphere to the shock along the ray θ = const, we have the following equation:

$$d\varepsilon / d\theta = -(1 + \varepsilon)\cot(\theta + \sigma) \tag{1.3}$$

The boundary conditions may be written as follows:

on the body $r = 1$,

$$u = 0, \quad \psi = 0, \quad \varphi = \varphi(0) = \text{const}$$

$$\left(\varphi(0) = \frac{2}{\varkappa+1}\frac{1}{w_+^{2\varkappa}}\left\{w_+^2 - \frac{(\varkappa^2-1)}{4\varkappa}\left(1-\frac{\varkappa-1}{\varkappa+1}w_+^2\right)\right\}\left(1-\frac{\varkappa-1}{\varkappa+1}w_+^2\right)^{-1}\right)$$

on the shock wave $r = 1 + \varepsilon(\theta)$,

$$u = w_y \sin\theta - w_x\cos\theta, \qquad w_x = w_+\cos(\sigma-\vartheta)\cos\sigma + \frac{\sin\sigma}{w_+\sin(\sigma-\vartheta)}\beta$$

$$v = w_x\sin\theta + w_y\cos\theta, \qquad w_y = w_+\cos(\sigma-\vartheta)\sin\sigma - \frac{\cos\sigma}{w_+\sin(\sigma-\vartheta)}\beta$$

$$P = \left[\frac{2w_+^2\sin^2(\sigma-\vartheta)}{\varkappa+1} - \frac{\varkappa-1}{2\varkappa}\left(1-\frac{\varkappa-1}{\varkappa+1}w_+^2\right)\right]\gamma, \qquad \rho = w_+^2\sin^2(\sigma-\vartheta)\,\frac{\gamma}{\beta} \qquad (1.4)$$

$$\beta = \left[1-\frac{\varkappa-1}{\varkappa+1}w_+^2\cos(\sigma-\vartheta)\right], \qquad \gamma = \left(1-\frac{\varkappa-1}{\varkappa+1}w_+^2\right)^{1/(\varkappa-1)}$$

Here σ is the angle of inclination of the shock wave to the axis of symmetry, and w_x and w_y are the velocity components along x and y.

2. Let us consider as an example the flow from a three-dimensional source past a sphere. Three-dimensional source flow occurs outside a sphere of radius r_*, on which the velocity reaches critical value, while the velocity outside the sphere for a supersonic source increases up to $M = \infty$ at infinity. Let us put the source at a distance C from the origin in the negative direction of the x-axis (Fig.1). The nonuniform flow just ahead of the shock is given by

$$\vartheta = \tan^{-1}\frac{(1+\varepsilon)\sin\theta}{C-(1+\varepsilon)\cos\theta}$$

$$w_+ = \frac{1}{d}\frac{1}{(1+\varepsilon)^2}\left(\frac{2}{\varkappa+1}\right)^{1/(\varkappa-1)}\frac{\sin^2\vartheta}{{}^1/_2\, w_+\sin^2\theta}\left(1-\frac{\varkappa-1}{\varkappa+1}w_+^2\right)^{-1/(\varkappa-1)} \qquad (2.1)$$

Here d is the ratio of the sphere radius to r_*.

System (1.2), (1.3) is solved by the method of integral relations. The region of integration is divided into strips equidistant in r. The integrand functions are represented by interpolating polynomials in r with interpolating node points at the boundaries of the strips. For the nth approximation, the system of equations may be written schematically as in [2]

$$\frac{d\varepsilon}{d\theta} = -(1+\varepsilon)\cot(\sigma+\theta), \qquad \frac{d\sigma}{d\theta} = F, \qquad \frac{du_i}{d\theta} = \Phi_i, \qquad \frac{dv_0}{d\theta} = \frac{F_0}{1-v_0^2}$$

$$\frac{dv_i}{d\theta} = F_i\left(w_i^2 - \frac{\varkappa+1-2u_i^2}{\varkappa-1}\right)^{-1}, \qquad \frac{d\psi_i}{d\theta} = r_i\rho_i\left(v_i\frac{dr_i}{d\theta} - r_iu_i\right)\sin\theta$$

$$\varphi_i = \varphi_1(\psi_i) \qquad (i = 2, 3, \ldots, n) \qquad (2.2)$$

Here F, F_0, F_i and Φ_i are the functions defined and holomorphic in the region of integration. System (2.2) is integrated numerically from the axis of symmetry $\theta = 0$, where $v_i = v_0 = 0$, $\psi_i = 0$, $\sigma = \frac{1}{2}\pi$, and the unknown parameters u_i and the value ε are determined from the requirement that the solution be regular at singular points. At each step of the integration, we take into account the parameters of the nonuniform flow in (2.1) and the derivatives $d\vartheta/d\theta$ and $dw_+/d\theta$ in the right-hand sides of (2.2).

3. We give some results of the numerical calculations, carried out in the first approximation in the region of subsonic and mixed flows.

Let M_0 denote the Mach number on the axis of symmetry just ahead of the

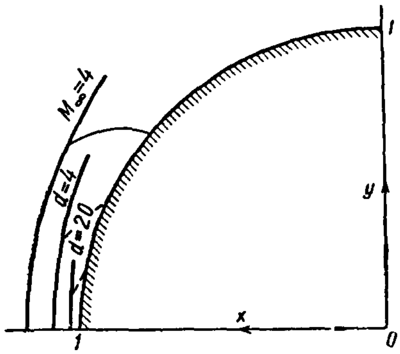

Fig. 2

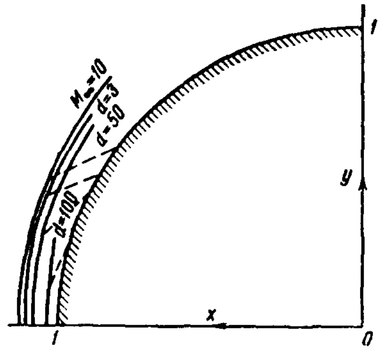

Fig. 3

shock wave. In Figs.2 and 3 are shown the shock waves and location of the sonic points on the sphere and shock for $M_0 = 4$ and 10.

For $M_0 = 4$ at $d = 4$ and 20, we have $\sigma = 2.28$ and 1,27; for $M_0 = 10$ at $d = 3$, 50 and 100, we have $\sigma = 11.47$, 3.15 and 1.39, respectively.

To compare the cases, we show the shock waves obtained for the uniform flow past a sphere with $M_\infty = 4$ and 10. Even a small nonuniformity in the angle results in a significant shift of the sonic points towards smaller θ and a decrease in the width of the shock layer. In all the cases studied, the sonic lines lie below the ray $\theta = \text{const}$ passing through the sonic point on the sphere.

In Fig.4 is shown the pressure distribution $P = P(\theta)/P(0)$ over the sphere for $M_0 = 4$ at different values of d.

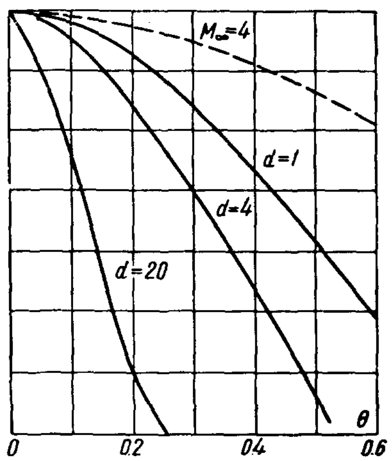

Fig. 4

The dashed curve shows the pressure for a uniform flow with $M_\infty = 4$.

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